647510 **MSC1PhyC003x** **Seat No :________**

M.Sc.(Phy.) Semester - 1 (*CBCS*) Examination
Jan/Feb.-2022 [NEW COURSE]
Quantum Mechanics I(Core (New))

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q - 1: Answer in brief any <u>SEVEN</u> **14**

(a) How the dimensionless coordinates $\in$ and $\boldsymbol{\xi}$ are defined?

(b) How the Hamiltonian, **H** can be represented in terms of "**a**" and "$\mathbf{a}^{\dagger}$" ?

(c) Prove: $[J_z, J_+] = \hbar J_+$

(d) Show that $[J_+, J_-] = 2\hbar J_z$

(e) Depict the classical and quantum probability curves for **n = 0** and **n = 1**.

(f) Why Fermi's rule is known as golden rule?

(g) Why WKB approximation is known as semi-classical approximation?

(h) What is trial wave function? How it is selected?

(i) What is the main use of Variation Method?

(j) Why matrix is used as operators in Quantum Mechanics?

Q - 2: Answer the following questions: Any <u>TWO</u> out of THREE **14**

(a) In obtaining the solution of SchrÖdinger equation for the Linear Oscillator, derive up to the below given equation –

$$\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + h(\epsilon - 1) = 0$$

(b) Explain Harmonic Oscillator Energy Spectrum.

(c) Discuss the Orthogonality of Hermite Polynomial and show that –

$$I_{m,n} = \int_{-\infty}^{+\infty} u_m(\xi)\, u_n(\xi) d\xi = 0 \ \text{ if } m \neq n$$

Q - 3: Answer the following questions: (any two) **14**

(a) What is Coordinate Transformation? Obtain the gradient operator, $\vec{\nabla}$ in spherical polar coordinates as –

$$\vec{\nabla} = \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \vec{e}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

(b) Using the concept of raising and lowering operators for angular momentum, obtain the following relations,

(i) $\lambda_J - m_{max}(m_{max} + 1)\hbar^2 = 0$

(ii) $\lambda_J - m_{min}(m_{min} - 1)\hbar^2 = 0$

(c) Prove that; in the time independent perturbation, the perturbation removes degeneracy and obtain the following relation for doubly degenerate states:

$W^{(1)} = \frac{1}{2}[h_{11}+h_{22}] \pm \frac{1}{2}[(h_{11}-h_{22})^2 + 4h_{12}h_{21}]^{1/2}$.

(d) Using ∇^2, obtain the SchrÖdinger equation in polar coordinates and by using the wave function $\Psi(r) = R(r)\,Y(\theta, \emptyset)$, separate the radial equation and derive the following equation,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}\left[E - V(r) - \frac{\lambda}{r^2}\right]R = 0$$

Q - 4: Answer the following questions: Any <u>TWO</u> out of THREE **14**

(a) Obtain the solution of simple harmonic oscillator using Variation Method.

Select trial wave function as $\psi = ce^{-\lambda x^2}$, where **C** and **λ** are constants.

(b) What is time dependent perturbation? Obtain the following general formulation as –

$$i\hbar\frac{dc_m(t)}{dt} = \sum_n c_n(t)e^{i(E_m-E_n)\frac{t}{\hbar}} < \Phi_m|H_1|\Phi >$$

(c) Solve the anharmonic oscillator problem using time independent perturbation theory, the Hamiltonian is given as –

$$H = \frac{p2}{2m} + \frac{1}{2}kx^2 + ax^3 + bx^4$$

and prove that energy eigen values are shifted to higher values, by the value of $\frac{3}{2}b\frac{\hbar\omega}{k^2}E_0^{(0)}$

Q - 5: Write any <u>TWO</u> short-notes on: **14**

(a) Spherical Harmonics

(b) The Raising, Lowering and Number Operators

(c) WKB Approximation

(d) Fermi golden rule

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